Water Renewal Time Scales in a Lake-River Chain System

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Flushing Time

\[ T_f = \frac{V}{Q} \]

Residence Time

Assumption: Continuous Stirred Tank Reactor (CSTR)

\[ \ln C(t) = -\left(\frac{1}{T_f}\right)t + \ln C_0 \]

Times

Spatial Domain

e-folding: 37%

Time to leave system

Age

Area of Interest

Time to Reach

\[ \text{Age } A < \text{Age } B \]
Lake Hydrodynamics
Lake Hydrodynamics

- Bathymetry/Morphometry
- Meteorological (Wind/Heat)
- Tributary

Water Level Circulation
River Dynamics

Free Flow

Dam Impoundment
Research Question

- What are time scales (*flushing, residence, age*) for Lakes & Rivers driven by these hydrodynamics?
Flush Time

RECALL:

\[ T_f = \frac{V}{Q} \]

YEAR AVERAGE DISCHARGE (cfs) RAINFALL (in) FLUSHING TIME (years)

<table>
<thead>
<tr>
<th>Year</th>
<th>Discharge (cfs)</th>
<th>Rainfall (in)</th>
<th>Flushing Time (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>135.8</td>
<td>28.76</td>
<td>4.16</td>
</tr>
<tr>
<td>2005</td>
<td>92.7</td>
<td>19.29</td>
<td>6.10</td>
</tr>
<tr>
<td>2006</td>
<td>113.6</td>
<td>21.28</td>
<td>4.98</td>
</tr>
<tr>
<td>2007</td>
<td>172.4</td>
<td>31.88</td>
<td>3.28</td>
</tr>
<tr>
<td>2008</td>
<td>258.5</td>
<td>25.54</td>
<td>2.19</td>
</tr>
<tr>
<td>2009</td>
<td>212.0</td>
<td>36.27</td>
<td>2.67</td>
</tr>
<tr>
<td>2010</td>
<td>184.1</td>
<td>29.63</td>
<td>3.07</td>
</tr>
<tr>
<td>2011</td>
<td>165.1</td>
<td>17.77</td>
<td>3.42</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>26.30</td>
<td>3.39</td>
</tr>
</tbody>
</table>

Average Discharge (cfs)

- 2004: 135.8
- 2005: 92.7
- 2006: 113.6
- 2007: 172.4
- 2008: 258.5
- 2009: 212.0
- 2010: 184.1
- 2011: 165.1

Average Discharge: 166.8

~ 3.5 Years
**Approach**

**1. Hydrodynamic model**
- Horizontal resolution: 100 meters
- Vertical layers: 52

**2. Particle Transport Model**

**Model Execution**
- AMD 64 processors (parallel computation)

**Model Setup**
- 8 Year Simulation
- Boundary Conditions
  - Wind Stress
  - Heat Flux
  - Inflow/Outflow
Horizontal Dispersion Test

- Switch off advection term and release 1,000 particles at same time from center
- Check to see that particles remain within 3 times of standard deviation $= \sqrt{6Kt}$

This means that 99.7% of total particles should be located within area
1. Hydrodynamic Model Calibration

![Map of Hydrodynamic Model](image1)

- **Surface**
  - Temperature (°C) over time for Surface observations and model predictions.

- **Bottom**
  - Temperature (°C) over time for Bottom observations and model predictions.

![Temperature Profiles](image2)

- Observed data compared with model predictions for deep hole temperature.

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**Temperature Contours**

- Contours of water column temperature.
  - Color scale from 5°C to 30°C.

- Depth range from 0 to 20 meters.

- Observations and model predictions for Jul.01 and Aug.01.
2. Particle Transport Model

\[ X^{n+1} = X^n + U \Delta t \]
2. Particle Transport Model

\[ X^{n+1} = X^n + U \Delta t + R \sqrt{2 K_x \Delta t} \]

**Advection**

**Dispersion**

Particle Depth (m)

Time (days)=0.041667
2. Particle Transport Model

\[ X^{n+1} = X^n + U \Delta t + R \sqrt{2 K_x \Delta t} \]

Advection

Dispersion

Particle Depth (m)

Advection-Dispersion
Stratification

Stratified

Unstratified

July - 01 12 AM

November - 01 12 AM

Depth (m)

Temperature (°C)

Depth (m)

Temperature (°C)

model

observed

observed
Wind Up/Down welling

Baroclinic

Barotropic
Flushing Time

Lake

Stratified

Unstratified

\[ C(t) = C_0 e^{-\left(\frac{Q}{V}\right)t} \]

\[ t/T_f \]

\[ \ln C(t) = -\left(\frac{1}{T_f}\right)t + \ln C_o \]

\[ T_f = \frac{V}{Q} \]

No. of Particles Remaining

Years

No. of Particles Remaining

Years

No. of Particles Remaining

Years
Flush Time Lake-River

2.2 Years

1.0 Years

1.2 Years

2.2 Years
Summary

1. Hydrodynamic Model
2. Particle Transport Model

**Flushing Time**

- **Lake**
  - 6.82 Years

- **Lake-River**
  - 2.2 Years
  - 1.2 Years

**Residence Time**

- **Lake**
  - 6.82 Years

- **Lake-River**
  - 2.2 Years
  - 1.2 Years

**Age**

- **Lake**
  - 14 DAYS

- **Lake-River**
  - 440 DAYS
  - 367 DAYS
Future Work

Optimization
Lock/Dam Management

Flooding

Quality

Water Level RESPONSE

η_{Mendota}  η_{Monona}  η_{Waubesa}  η_{Kegonsa}

Q_{MM}  Q_{MW}  Q_{WK}  Q_{KS}